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Math 12 Honours: Section 5.1 Solving Exponential Functions

1. Solve for all values of "x" :

a) $8^{x+1}(32)=128$

$2^{3x+3}\cdot 2^5 = 2^7$

$3x+8=7$

$x=-\frac{1}{3}$

b) $\left(27^{2x}\right)^3=\frac{1}{729}$

$(3^{6x})^3 = 3^{-6}$

$18x=-6$

$x=-\frac{1}{3}$

c) $\frac{8^{-1}+2^{-3}}{64^x}=\left(2^{2x-3}\right)$

$\frac{\frac{1}{4}}{64^x} = 2^{2x-3}$

$\frac{2^{-2}}{2^{6x}} = 2^{2x-3}$

$-2-4x = 2x-3$

$x=\frac{1}{6}$

d) $\frac{256^x}{4^{2x-6}}=\left(2^{2x}\right)^x$

$\frac{2^{8x}}{2^{4x-12}} = 2^{2x^2}$

$8x-(4x-12) = 2x^2$

$4x+12 = 2x^2$

$2x^2-4x-6=0$

$x^2-2x-6=0$

$x=\frac{2\pm\sqrt{4+24}}{2}$

$=\frac{2\pm2\sqrt{7}}{2} = 1\pm\sqrt{7}$

$x_1 = 1+\sqrt{7}$

$x_2 = 1-\sqrt{7}$

e) $\left(8^{x+4}\right)^x=128^{2x+3}$

$(2^{3x+12})^x = 2^{14x+21}$

$2^{3x^2+12x} = 2^{14x+21}$

$3x^2+12x = 14x+21$

$3x^2-2x-21=0$

$(3x+7)(x-3)=0$

$x=3$ or $x=-\frac{7}{3}$

f) $\frac{\left(729^x\right)}{3\times 9^{2x-3}}=\left(27^{2x+3}\right)^x$

$\frac{3^{6x}}{3^{4x-5}} = 3^{6x^2+9x}$

$6x-(4x-5) = 6x^2+9x$

$2x+5 = 6x^2+9x$

$6x^2+7x-5=0$

$x=\frac{-7\pm\sqrt{49+120}}{12}$

$x=\frac{-7\pm13}{12}$

$x_1=\frac{1}{2}$ $x_2=-\frac{5}{3}$

2. Use Logarithms to solve for "x":

a) $4^{x-3}=10$ $(x-3)\log_{10}4 = 1$ $x = \frac{1}{\log 4}+3$ $x = \log_4 10 + 3$ $x = \frac{1}{2}\log_2 10 + 3$ $x = \frac{1}{2}\log_2 5 + 3.5$	b) $6^{2x+1}=14$ $(2x+1)\log_{14}6 = 1$ $2x+1 = \frac{\log 14}{\log 6}$ $2x = \frac{\log 14 - \log 6}{\log 6}$ $2x = \frac{\log \frac{7}{3}}{\log 6}$ $x = \frac{\log\frac{7}{3}}{\log 36}$ $x = \log_{36}\frac{7}{3}$
c) $9^{x^2}=20$ $x^2\log_{20}9 = 1$ $x^2 = \frac{\log 20}{\log 9}$ $x = \pm(\log_9 20)^{\frac{1}{2}}$	d) $10^{3-x}=21$ $(3-x)\log_{21}10 = 1$ $3-x = \frac{\log 21}{\log 10}$ $x = 3-\log_{10}21$ $x = 3-\log 21$
e) $(2^{2x})3^x = 8000$ $\log_2(2^{2x}\cdot 3^x) = \log_2(2^6\times 125)$ $2x + x\log_2 3 = 6+\log_2 125$ $x = \frac{2+\log_2 3}{6+\log_2 125}$	f) $6^{x^3}=20$ $x^3\log_{20}6 = 1$ $x^3 = \frac{\log 20}{\log 6}$ $x = \sqrt[3]{\frac{\log 20}{\log 6}}$ $x = (\log_6 20)^{\frac{1}{3}}$

3. Rewrite each of the following in logarithm form:

$a)\ e^5 = y$ $\ln y = 5$	$b)\ 2a^b = c$ $\log_a c - \log_a 2 = b$	$c)\ 2(3x)^{15} = y$ $\log_{3x} y - \log_{3x} 2 = 15$

4. Evaluate each of the following without a calculator:

$a)\log_5 125 = 3$	$b)\log_2 128 = 7$	$c)\log_9 2183 = \frac{7}{2}$
$d)\log_{16} 64 = \frac{3}{2}$ $\frac{1}{4}\ l\ 2^6$	$e)\log_7\sqrt{343} = \frac{3}{2}$ $\frac{1}{2}\log_7 7^3$	$f)(\log_{128}1024)^{-1} = \frac{7}{10}$ $(\frac{10}{7})^{-1}$
$g)\log_8\left(\frac{1}{4}\right) = -\frac{2}{3}$ -2 $\quad 2^{-2}$	$h)-\log_{2.5}\left(\frac{125}{8}\right)$ $= -\frac{\log\frac{125}{8}}{\log 2.5} = \frac{-\log(\frac{5}{2})^3}{\log(\frac{5}{2})} = -3$	$i)\left(\log_{0.125}64^{\frac{2}{3}}\right)^{-2} = \frac{1}{144}$ $(-3\log_2 2^4)^{-2} = (-12)^{-2} =$

5. Simplify each of the following logarithms without using a calculator:

a) $\log_6 24+\log_6 9$ $= \log_6 24\times 9$ $= 3$	b) $\log_5 100-\log_5 4$ $= \log_5 25$ $= 2$	c) $\log_4 8+\log_4 64$ $= \frac{3}{2}+\frac{6}{2}$ $= \frac{9}{2}$

d) $\log_3 324 - \log_3 4$ $= \log_3 81$ $= 4$	e) $\log_4 12 + \log_4\left(\frac{2}{3}\right) + 3\log_4 2$ $= \log_4 4 + \log_4 3 + \log_4 2 - \log_4 3$ $+ 3\log_4 2$ $= 1 + 2$ $= 3$	f) $\log_8 24 + 2\log_8 2 - \log_8 3$ $= \log_8 \frac{24 \times 4}{3}$ $= \log_8 32$ $= \frac{5}{3}$
g) $\log_3 \sqrt{45} - 0.5\log_3 5 + \log_3 9$ $= \frac{1}{2}\log_3 45 - \frac{1}{2}\log_3 5 + \log_3 9$ $= \frac{1}{2}\log_3 9 + \cancel{\frac{1}{2}\log_3 5} - \frac{1}{2}\log_3 5 + \log_3 9$ $= \frac{3}{2} \times 2$ $= 3$	h) $\log_3 5! - \log_3 4 - 1$ $= \log_3 \frac{5 \times 4 \times 3 \times 2}{4 \times 3}$ $= \log_3 10$	i) $\log_{x+1}\left(x^3 + 3x^2 + 3x + 1\right)$ $= \log_{x+1} (x+1)^3$ $= 3$

6. Solve for all possible values of "x" : $3^{x-1}\left(9^{\frac{3}{2x^2}}\right) = 27$

$3^{x-1} \cdot 3^{\frac{3}{x^2}} = 27$

$3^{x-1+\frac{3}{x^2}} = 3^3$

$x-1+\frac{3}{x^2} = 3$

$x^3 - x^2 + 3 = 3x^2$

$x^3 - 4x^2 + 3 = 0$

$x = 1$

$(x-1)(x^2-3x-3) = 0$

$x = 1$

or

$x^2 - 3x - 3 = 0$

$x = \frac{3 \pm \sqrt{21}}{2}$

7. Suppose you have the equation $2^{5x-3} = -1$, is it possible to have a solution? Explain and justify your answer:

assume $5x-2 = A$

if $A > 0$

$2^A > 0$

if $A = 0$

$2^0 = 1$

if $A < 0$

$2^A = \frac{1}{2^{-A}} > 0$

it's impossible for 2 to raise to a negative number

8. Given the equation, $m^x = m^y$, how many cases are there such that the equation is true?

(1) $x = y$

(2) $m = 0$ $x \neq 0$ $y \neq 0$

(3) $m = 1$

(4) $m = -1$ x is even y is even or x is odd y is odd.

9. Find all values of "x" such that: $6^2\left(6^x\right)^x = \left(6^x\right)\left(6^x\right)\left(6^x\right)$

$6^2 \cdot 6^{x^2} = 6^{3x}$

$6^{x^2+2} = 6^{3x}$

$x^2 + 2 = 3x$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1$ or $x = 2$

10. Solve for all possible values of "x": $4\times9^x-21=25\times3^x$

$3^x = A$

$4\times(3^x)^2-21=25\times3^x$

$4A^2-21=25A$

$4A^2-25A-21=0$

$(4A+3)(A-7)=0$

$A=7 \quad (A>0)$

$3^x=7$

$x=\log_3 7$

11. Solve for "x": $2^{x+1}+2^x=2^3+2^5+2^{7-x}$

$2^x\cdot2+2^x=2^3+2^5+\frac{2^7}{2^x}$

$A=2^x$

$2A+A=2^3+2^5+\frac{2^7}{A}$

$3A=2^3+2^5+\frac{2^7}{A}$

$3A^2=2^3A+2^5A+2^7$

$3A^2-40A-128=0$

$(3A+8)(A-16)=0$

$A=16 \quad (A>0)$

$2^x=16$

$x=4$

12. Suppose that $P=2^m$ and $Q=3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m,n)?

a) P^2Q b) P^nQ^m c) P^nQ^{2m} d) $P^{2m}Q^n$ e) $P^{2n}Q^m$ (AMC)

a)

$P^2=2^{2m}=4^m$

P^2Q

13. Solve the equation for "y" in terms of "x". For hat values of "x" is there no value of "y" that satisfies the equation: $(x+2)^{2y}-x^2=3$

$(x+2)^{2y}=3+x^2$

$2y\log(x+2)=\log(x^2+3)$

$2y=\frac{\log(x^2+3)}{\log(x+2)}$

$y=\frac{\log(x^2+3)}{2\log(x+2)}$

$y=\frac{\log(x^2+3)}{\log(x^2+4x+4)}$

$x>-2$

so for $x\le-2$

no y satisfies the equation

14. Suppose that $60^a=3$ and $60^b=5$. What is the value of $12^{\frac{1-a-b}{2-2b}}$?AMC12

$60^{a+b}=15 \Rightarrow \log_{60}15=a+b$

$60^{2b}=25 \Rightarrow \log_{60}25=2b$

$\log_2 12=a$

$(2^a)^{\frac{1}{a}}=(12)^{\frac{1}{a}}$

$12^{\frac{1-\log_{60}15}{2-\log_{60}25}}=12^{\frac{\log_{60}4}{\log_{60}144}}=12^{\frac{\log_2 4}{\log_2 144}}=12^{\frac{1}{\log_2 12}}=2$

15. Suppose "a", "b", "c" and "d" are positive integers, then what is the smallest possible value of $a+b+c+d$?

$4^a+4^b+4^c+4^d=4^{1023}$

suppose $a\ge b\ge c\ge d$

$4^a(1+4^{b-a}+4^{c-a}+4^{d-a})=4^{1023}$

$(1+4^{b-a}+4^{c-a}+4^{d-a})=4^{1}$

so $a=b=c=d$

$4^{a+1}=4^{1023}$

$a=1022$

$a+b+c+d=4088$

16. Given the equations below, find all the ordered triples of real values (a,b,c) that satisfy the them:

$$c^a = b^{2a}, \quad 2^c = 2\left(4^a\right), \quad a+b+c=10$$

$$\begin{cases} c^a = (b^2)^a \\ 2^c = 2^{2a+1} \end{cases}$$

① $\begin{cases} 2a+1 = c \\ c = b^2 \end{cases}$

② $c = b^2 = 1$ $\begin{cases} 2a+1 = c \\ c = 1 \\ b = \pm 1 \end{cases}$

③ $c = b^2 = 0$ $\begin{cases} c = 0 \\ b = 0 \\ 2a+1 = c \end{cases}$

① $2a+1 = c$

$c = b^2$

$a+b+c = 10$

$$\frac{b^2-1}{2} + b + b^2 = 10$$

$$b^2 - 1 + 2b + 2b^2 = 20$$

$$3b^2 + 2b - 21 = 0$$

$$(3b-7)(b+3) = 0$$

$$b = \frac{7}{3} \text{ or } b = -3$$

$$c = \frac{49}{9} \text{ or } c = 9$$

$\begin{cases} b = \frac{7}{3} \\ c = \frac{49}{9} \\ a = \frac{20}{9} \end{cases}$ $\begin{cases} b = -3 \\ c = 9 \\ a = 4 \end{cases}$

② $\begin{cases} a = 0 \\ b = 1 \\ c = 1 \end{cases}$ $\begin{cases} a = 0 \\ b = -1 \\ c = 1 \end{cases}$

③ $\begin{cases} a = -\frac{1}{2} \\ b = 0 \\ c = 0 \end{cases}$

but $a+b+c \neq 10$

So ② and ③ are impossible.

Summary: $\begin{cases} a = \frac{20}{9} \\ b = \frac{7}{3} \\ c = \frac{49}{9} \end{cases}$ or $\begin{cases} a = 4 \\ b = -3 \\ c = 9 \end{cases}$

Name:______________________ Date:______________________

Math 12 Honours: Section 5.2 What are Logs and Basic with Logarithm

$\log(A\times B)=\log A+\log B$ $\qquad \log\left(\frac{A}{B}\right)=\log A-\log B$ $\qquad \log A^n = n\log A$ $\qquad \log_a b^c = \frac{\log b^c}{\log a}$

1. Rewrite each of the following in exponential form: $\log(1-$

a) $\log_3 81 = 4$ $3^4 = 81$	b) $\log_4 2048 = b$ $4^b = 2048$ $(b = \frac{11}{2})$	c) $-\log_5 a = 13$ $5^{-13} = a$
d) $\log_c d = e$ $d = c^e$	e) $\log_{2x} 100 = 5$ $(2x)^5 = 100$	f) $\log_{\sqrt{3}} a = b$ $(\sqrt{3})^b = a$

2. Evaluate each of the following without using a calculator:

a) $\log_2 8^3$ $= 9$	b) $\log_3 \sqrt{243}$ $= \log_3 3^{\frac{5}{2}}$ $= \frac{5}{2}$	c) $\log 100 + \log_3 \sqrt{3}$ $= 2 + \frac{1}{2}$ $= \frac{5}{2}$
d) $3\log_{0.125} 32$ $= 3\times\frac{1}{-3}\log_2 32$ $= -5$	e) $\log_2\left(\log(0.0001)\right)^2$ $= \log_2(\log_{10}10^{-4})^2$ $= \log_2 16$ $= 4$	f) $\log_8\left(\frac{\sqrt{256}\times 64}{\sqrt[3]{1024}}\right)$ $= \log_{2^3}\left(\frac{2^4\times 2^6}{2^{\frac{10}{3}}}\right)$ $= \frac{1}{3}\log_2 2^{\frac{20}{3}} = \frac{20}{9}$
g) $\log_5 25\times\log_5 0.2\times\log_{0.2}125$ $= 2\times(-1)\times(-3)$ $= 6$	h) $\log_{12}16+\log_{12}9$ $= \log_{12}16\times 9$ $= 2$	i) $\log_{11}\left(2\log_4 2^{5!}+\log 10\right)$ $= \log_{11}(5!+1)$ $= \log_{11}(121)$ $= 2$

3. Expand and rewrite each expression in terms of log A, log B, and or log C

a) $\log\left(\frac{AB}{C}\right)$ $= \log A + \log B - \log C$	b) $\log\left(A^3\sqrt{B}\times C^{-1}\right)$ $= \log A^3 + \log\sqrt{B} + \log C^{-1}$ $= 3\log A + \frac{1}{2}\log B - \log C$	c) $\log\left(\frac{10A}{\sqrt{B}}\right) - 0.5\log(100C)$ $= \log 10A - \log\sqrt{B} - 0.5\log 100C$ $= 1 + \log A - \frac{1}{2}\log B - 1 - 0.5\log C$

$= \log A - \frac{1}{2}\log B - \frac{1}{2}\log C$

d) $\log\left(0.001\sqrt{ab}\right)$

$= \log 10^{-3} + \frac{1}{2}\log ab$

$= -3 + \frac{1}{2}\log a + \frac{1}{2}\log b$

e) $\log\left(\dfrac{100a}{\sqrt{c^3}b^4}\right)$

$= \log 100a - \log \sqrt{c^3}b^4$

$= 2 + \log a - \log c^{\frac{3}{2}} - \log b^4$

$= 2 + \log a - \frac{3}{2}\log c - 4\log b$

f) $\log\left(\dfrac{\sqrt{10b}}{a^{3/2}}\right) - \log c^{-1}$

$= \log 10^{\frac{1}{2}} + \log b^{\frac{1}{2}} - \log a^{\frac{3}{2}} + \log c$

$= \frac{1}{2} + \frac{1}{2}\log b - \frac{3}{2}\log a + \log c$

g) $\log\left(\dfrac{a^{-2}}{1000b^2}\right)$

$= \log a^{-2} - \log 1000b^2$

$= -2\log a - 3 - 2\log b$

h) $\left[\log_c\left(\dfrac{a^2}{b^3}\right)\right]^{-1}$

$= [\log_c a^2 - \log_c b^3]^{-1}$

$= [2\log_c a - 3\log_c b]^{-1}$

$= \dfrac{1}{2\log_c a - 3\log_c b}$

$= \dfrac{\log c}{2\log a - 3\log b}$

i) $\dfrac{\log_a b}{\log_c a} \times \dfrac{\log_b ac}{\log_b c}$

$= \dfrac{\log b}{\log a} \div \dfrac{\log a}{\log c} \times \dfrac{\log ac}{\cancel{\log b}}$

$= \dfrac{\log b}{\log a} \times \dfrac{\log c}{\log a} \dfrac{\log a + \log c}{\log c} \dfrac{\log c}{\cancel{\log b}}$

$= \dfrac{\log a(\log a + \log c)}{\log^2 a}$

4. Express each of the following as a single logarithm

a) $2\log a + 5\log b$

$= \log a^2 + \log b^5$

$= \log a^2 b^5$

b) $3\log x + \frac{1}{2}\log y$

$= \log x^3 + \log \sqrt{y}$

$= \log x^3\sqrt{y}$

c) $2\log a + \log b - 5\log c$

$= \log a^2 + \log b - \log c^5$

$= \log \dfrac{a^2 b}{c^5}$

$= \dfrac{\log b}{\log a} + \dfrac{\log b \log}{\log^2 a}$

d) $\frac{1}{2}\log x - \frac{2}{3}\log y$

$= \log \sqrt{x} - \log \sqrt[3]{y^2}$

$= \log \dfrac{\sqrt{x}}{\sqrt[3]{y^2}}$

e) $3\log a - 4\log b - 0.5\log c$

$= \log a^3 - \log b^4 - \log \sqrt{c}$

$= \log \dfrac{a^3}{b^4\sqrt{c}}$

f) $10\log a - \dfrac{3\log b}{2}$

$= \log a^{10} - \log b^{\frac{3}{2}}$

$= \log \dfrac{a^{10}}{b^{\frac{3}{2}}}$

f) $\dfrac{2\log a}{3\log b} + \dfrac{5\log b}{2\log b}$

$= \dfrac{4\log a}{6\log b} + \dfrac{15\log b}{6\log b}$

$= \dfrac{\log a^4 b^{15}}{6\log b}$

g) $\dfrac{\log a}{0.4\log c} + \dfrac{\log b}{0.5\log c}$

$= \frac{5}{2}\log_c a + 2\log_c b$

$= \log_c a^{\frac{5}{2}} b^2$

h) $3\log\sqrt{a} + 2\log\sqrt[3]{b} - 3$

$= \log a^{\frac{3}{2}} + \log b^{\frac{2}{3}} - \log 10^3$

$= \log \dfrac{a^{\frac{3}{2}} b^{\frac{2}{3}}}{1000}$

5. If $\log_3 2 = x$, simplify each logarithm in terms of "x"

$a)\ \log_3 8$

$= 3\log_3 2$

$= 3x$

$b)\ \log_3 \sqrt{2}$

$= \frac{1}{2}\log_3 2$

$= \frac{x}{2}$

$c)\ \log_3 24$

$= \log_3 3 + 3\log_3 2$

$= 1 + 3x$

$d)\ \log_3 18\sqrt{8}$

$= \log_3 2 \times 9 \times 2^{\frac{3}{2}}$

$= \log_3 2^{\frac{5}{2}} \times 9$

$= 2 + \frac{5}{2}x$

6. Given $\log_3 x = 2$ and $\log_3 y = 5$, evaluate each logarithm

$a)\log_3 xy$

$= \log_3 x + \log_3 y$

$= 2 + 5$

$= 7$

$b)\log_3\left(27x^3y\right)$

$= \log_3 27 + 3\log_3 x + \log_3 y$

$= 3 + 3\times 2 + 5$

$= 14$

$c)\log_3\left(\frac{3x}{y^3}\right)$

$= \log_3 3x - \log_3 y^3$

$= 1 + \log_3 x - 3\log_3 y \quad = 1 + 2 - 3\times 5 = -12$

$d)\log_3\left(810x^{-2}y\right)$

$= \log_3 \frac{810}{x^2} \cdot y$

$= \log_3 810 + \log_3 y - 2\log_3 x$

$= 5 - 2\times 2 + \log_3 810$

$= 1 + 4 + \log_3 10 \quad = 5 + \log_3 10$

7. Use the fact that $\log_a b = \frac{\log_x b}{\log_x a}$ to simplify the following:

$a)\ (\log_x y)(\log_y x)$

$= \frac{\log y}{\log x} \cdot \frac{\log x}{\log y}$

$= 1$

$b)\ (\log_5 8)(\log_8 7)(\log_7 5)$

$= \frac{\log 8}{\log 5} \cdot \frac{\log 7}{\log 8} \cdot \frac{\log 5}{\log 7}$

$= 1$

$c)\ \left(\frac{1}{\log_d x}\right)\left(\frac{1}{\log_c x}\right)$

$= \frac{1}{\frac{\log x}{\log d}} \cdot \frac{1}{\frac{\log x}{\log c}}$

$= \frac{\log d \cdot \log c}{\log x \cdot \log x}$

$d)\ (\log_4 a)(\log_a 2a)(\log_{2a} x)$

$= \frac{\log a}{\log 4} \cdot \frac{\log 2a}{\log a} \cdot \frac{\log x}{\log 2a}$

$= \log_4 x$

$e)\ \dfrac{\log_x a}{\log_{xy} a}$

$= \dfrac{\frac{\log a}{\log x}}{\frac{\log a}{\log xy}} = \dfrac{\log x + \log y}{\log x} = 1 + \log_x y$

$f)\ \dfrac{\log_c m}{\log_{cd} m} - \dfrac{\log_c m}{\log_d m}$

$= \dfrac{\frac{\log m}{\log c}}{\frac{\log m}{\log c + \log d}} - \dfrac{\frac{\log m}{\log c}}{\frac{\log m}{\log d}} = \dfrac{\log c + \log d}{\log c} - \dfrac{\log d}{\log c}$

$= 1$

8. Solve for "y" $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$

$\dfrac{\cancel{\log x}}{\log 3} \cdot \dfrac{\cancel{\log 2x}}{\cancel{\log x}} \cdot \dfrac{\log y}{\cancel{\log 2x}} = \dfrac{2\cancel{\log x}}{\cancel{\log x}}$

$\dfrac{\log y}{\log 3} = 2$

$\log y = \log 9$

$y = 9$

9. If $\log x^5 y^3 = 25$ and $\log \dfrac{x}{y} = 3$, then what is the value of $\log x$?

$\log x^5 y^3 = 25$

$\begin{cases} 5\log x + 3\log y = 25 \\ \log x - \log y = 3 \end{cases}$

$\begin{cases} 5\log x + 3\log y = 25 \\ 3\log x - 3\log y = 9 \end{cases}$

$8\log x = 34$

$\log x = \dfrac{17}{4}$

10. Prove the following identity: $\dfrac{1}{\log_a b} + \dfrac{1}{\log_c b} = \dfrac{1}{\log_{ac} b}$

$\dfrac{1}{\log_a b} + \dfrac{1}{\log_c b} = \dfrac{\log a}{\log b} + \dfrac{\log c}{\log b} = \dfrac{\log a + \log c}{\log b} = \dfrac{\log ac}{\log b} = \dfrac{1}{\frac{\log b}{\log ac}} = \dfrac{1}{\log_{ac} b}$

11. Find the value of $\sum_{n=1}^{999} \log \sqrt[3]{\dfrac{n^2}{n^2+2n+1}}$

$= \sum_{n=1}^{999} \log \left(\dfrac{n^2}{n^2+2n+1}\right)^{\frac{1}{3}} = \dfrac{1}{3}\sum_{n=1}^{999} \log \dfrac{n^2}{(n+1)^2} = \dfrac{2}{3}\sum_{n=1}^{999} \log \dfrac{n}{n+1}$

$= \dfrac{2}{3}\left(\log \dfrac{1}{2} + \log \dfrac{2}{3} + \cdots + \log \dfrac{999}{1000}\right)$

$= \dfrac{2}{3}(\log 1 - \log 2 + \log 2 - \log 3 + \cdots + \log 999 - \log 1000)$

$= \dfrac{2}{3}(\log 1 - \log 1000)$

$= \dfrac{2}{3}(0 - 3)$

$= -2$

12. Simplify the product completely: $\dfrac{\log_2 3}{\log_4 3} \times \dfrac{\log_4 5}{\log_6 5} \times \dfrac{\log_6 7}{\log_8 7} \times \ldots \times \dfrac{\log_{124} 125}{\log_{126} 125} \times \dfrac{\log_{126} 127}{\log_{128} 127}$

$\dfrac{\frac{\log 3}{\log 2}}{\frac{\log 3}{\log 4}} \times \dfrac{\frac{\log 5}{\log 4}}{\frac{\log 5}{\log 6}} \cdots \times \dfrac{\frac{\log 127}{\log 126}}{\frac{\log 127}{\log 128}}$

$= \dfrac{\log 4}{\log 2} \times \dfrac{\log 6}{\log 4} \times \cdots \times \dfrac{\log 128}{\log 126}$

$= \dfrac{\log 128}{\log 2} = 7$

13. If $\log(xy^3)=1$ and $\log(x^2y)=1$. What is $\log(xy)$? Amc 2003 12A

$\log x + 3\log y = 1 \Rightarrow 2\log x + 6\log y = 2$

$2\log x + \log y = 1$

$5\log y = 1$ $\quad y^5 = 10$ $\quad y = 10^{\frac{1}{5}}$

$2\log x = \frac{4}{5}$ $\quad \log x = \frac{2}{5}$

$\log(xy) = \log x + \log y = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

14. If $a \ge b > 1$, what is the largest possible value of $\log_a(a/b) + \log_b(b/a)$? AMC 2003 12B

$\log_a \frac{a}{b} + \log_b \frac{b}{a} = \log_a a - \log_a b + \log_b b - \log_b a$

$= 2 - \log_a b - \log_b a$

$2 - \log_a b - \log_b a$

$(\log_a b)(\log_b a) = 1 \quad xy = 1$ $\quad$ AM-GM $\frac{x+y}{2} \ge \sqrt{xy}$ $\quad x+y \ge 2\sqrt{xy} \Rightarrow x+y \ge 2$

$(x+y)_{min} = ?$

15. How many distinct four-tuples (a,b,c,d) of rational numbers are there with:

$a\log_{10}2 + b\log_{10}3 + c\log_{10}5 + d\log_{10}7 = 2005$ Amc 2005 12B

$\log_{10} 2^a + \log_{10} 3^b + \log_{10} 5^c + \log_{10} 7^d = 2005$

$\log_{10} 2^a 3^b 5^c 7^d = 2005 = \log_{10} 10^{2005} = \log_{10} 2^{2005} 5^{2005}$

$a = 2005$, $b = 0$, $c = 2005$, $d = 0$ $\quad$ only 1 triple ?

16. The numbers $\log(a^3b^7)$, $\log(a^5b^{12})$, $\log(a^8b^{15})$ are the first three terms of an arithmetic sequence, and the 12th term of the sequence is $\log(b^n)$. what is the value of "n"? amc 12

$\log(a^3b^7) = 3\log a + 7\log b$

$\log(a^5b^{12}) = 5\log a + 12\log b$

$\log(a^8b^{15}) = 8\log a + 15\log b$

$10\log a + 24\log b = 11\log a + 22\log b$

$\log a = \log b^2$

$\log(a^3b^7) = \log b^{13} = 13\log b$

$\log(a^5b^{12}) = \log b^{22} = 22\log b$

$\log(a^8b^{15}) = \log b^{31} = 31\log b$

Term $12^{th} = 13\log b + 9\log b(12-1) = 112\log b$

$n = 112$

17. Two distinct numbers "a" and "b" are chosen randomly from the set $\{2, 2^2, 2^3, \ldots\ldots 2^{26}\}$. What is the probability that $\log_a b$ is an integer? AMC 12 modified

$a = 2^m \quad b = 2^n$

$\log_{2^m} 2^n = k$ $\quad$ k is an integer

$\frac{n}{m}\log_2 2 = k$

$k = \frac{n}{m}$

$n > m \quad m \mid n$

		# of m
n = 26	= 2×13	3
n = 25	= 5²	2
n = 24	= 2³×3	7
n = 23	= 23	1
n = 22	= 2×11	3
n = 21	= 3×7	3
n = 20	= 2²×5	5
n = 19		1
n = 18	= 2×3²	5
n = 17	= 17	1
n = 16	= 2⁴	4
n = 15	= 3×5	3
n = 14	= 2×7	3
n = 13	= 13	1

Total # of (m) = 3+2+7+1+3+3+5+1+5+1+4+3+3+1 = 42

Probability $= \frac{42}{{}_{26}P_2} = \frac{42}{26 \cdot 25} = \frac{42}{650} = \frac{21}{325}$

Name: Siyue Zhang Date: ____________

Math 12 Honours: Section 5.3 Solving Logarithmic Equations with Identities

1. Solve for "x" and state all the extraneous roots: Show your work:

a) $\log(6-x)-2\log x=0$

$6-x=x^2$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3$

or

$x=2$

$x>0$ $x=2$ $x\neq-3$ (ext.)

b) $\log_3 x^3-\log_3 3x=3$ $\log_3 27$

$\frac{x^3}{3x}=27$

$x^2=81$

$x=\pm 9$

$x>0$

$x\neq -9$ ext. $x=9$

c) $\log_2(x-3)+\log_2(x+1)=5$

$(x-3)(x+1)=32$

$x^2-2x-3-32=0$

$x^2-2x-35=0$

$(x-7)(x+5)=0$

$x=7$ or $x=-5$

$x-3>0$

$x\neq-5$ ext. $x=7$

d) $\log_7(x+1)+\log_7 x=\log_7 12$

$x^2+x=12$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4$ or $x=3$

$x>0$

$x\neq-4$ ext. $x=3$

e) $\log_4(16x-64)-\log_4(3x-34)=3$

$\frac{16x-64}{3x-34}=64$

$\frac{x-4}{3x-34}=16$

$x-4=48x-34\times16$

$34\times16-4=47x$

$x=\frac{540}{47}$

f) $\log(x-7)+\log(x+2)=1$

$(x-7)(x+2)=10$

$x^2-5x-14-10=0$

$x^2-5x-24=0$

$(x-8)(x+3)=0$

$x=8$

or

$x=-3$ $x-7>0$ $x\neq-3$ ext.

g) $\log_3(3x+2)+\log_3 x=\log_3 56$

$(3x+2)x=56$

$3x^2+2x-56=0$

$(3x+14)(x-4)=0$

$x=-\frac{14}{3}$ or $x=4$

$x>0$ $x\neq-\frac{14}{3}$ ext.

$x=4$

h) $\log x=\frac{2}{3}\log 27+2\log 2-\log 3$

$\log x=\log 9+\log 4-\log 3$

$\log x=\log 12$

$x=12$

i) $2\log_4 x+\log_4(x-2)-\log_4 2x=1$

$\log_4 \frac{x^2(x-2)}{2x} = 1$

$\frac{x(x-2)}{2} = 4$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4$ or $x=-2$

$x>0$ $x\neq -2$ (ext.) $x=4$

j) $\log_2 16^{2x+1}=8$

$\log_2 2^{8x+4} = 8$

$8x+4=8$

$x=\frac{1}{2}$

l) $(\log_8 a)(\log_a 3a)(\log_{3a} x^2)=\log_a a^5$

$\frac{\cancel{\log a}}{3\log 2} \cdot \frac{\cancel{\log 3a}}{\cancel{\log a}} \cdot \frac{2\log x}{\cancel{\log 3a}} = 5$

$2\log x = 15\log 2$

$x^2 = 2^{15}$

$x = \pm 128\sqrt{2}$

$x>0$ $x\neq -128\sqrt{2}$ (ext.)

$x=128\sqrt{2}$

m) $2\log(3-x)=\log 2+\log(22-2x)$

$\log(3-x)^2 = \log(22-2x)\cdot 2$

$x^2-6x+9 = 44-4x$

$x^2-2x-35=0$

$(x-7)(x+5)=0$

$x=7$ or $x=-5$

$3-x>0$ $x\neq 7$ (ext.)

$x=-5$

n) $2^{\log x^2}=3\left(2^{1+\log x}\right)+16$

$2^{2\log x} = 3(2\times 2^{\log x})+16$

$A=2^{\log x}$

$A^2 = 3(2\times A)+16$

$A^2-6A-16=0$

$(A-8)(A+2)=0$

$A=8$ $A=-2$

$2^{\log x}>0$

$A\neq -2$ (ext.)

$2^{\log x}=8$

$\log x = 3$

$x=1000$

o) $\log_5(x+3)+\log_5(x-1)=1$

$\log_5[(x+3)(x-1)] = \log_5 5$

$x^2+2x-3=5$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=2$ or $x=-4$

$x-1>0$

$x\neq -4$ (ext.) $x=2$

$\log_2(x+2)+5=8+\log_2 x$

$\log_2(x+2)-\log_2 x = 3$

$\log_2\frac{x+2}{x} = \log_2 8$

$\frac{x+2}{x} = 8$

$x+2 = 8x$

$x=\frac{2}{7}$

$2\log x-\log(24-x)=\log 2$

$\log\frac{x^2}{24-x} = \log 2$

$\frac{x^2}{24-x} = 2$

$x^2 = 48-2x$

$x^2+2x-48=0$

$(x+8)(x-6)=0$

$x=6$ or $x=-8$

$x>0$ $x\neq -8$ (ext.)

$x=6$

$\log_2(2x+4)-\log_2(x-1)=3$ $\log_2 \frac{2x+4}{x-1} = \log_2 8$ $\frac{2x+4}{x-1} = 8$ $2x+4 = 8x-8$ $6x = 12$ $x = 2$	$\log_4 x+\log_2\sqrt{x-2}=1+\log_{16}(x-1)^2$ $\frac{1}{2}\log_2 x + \frac{1}{2}\log_2(x-2) = 1+\frac{2}{4}\log_2(x-1)$ $\log_2 x + \log_2(x-2) = 2+\log_2(x-1)$ $\log_2(x(x-2)) = \log_2 4(x-1)$ $x^2-2x = 4x-4$ $x^2-6x+4=0$ $x = \frac{6\pm\sqrt{36-16}}{2} = \frac{6\pm\sqrt{20}}{2}$ $= 3\pm\sqrt{5}$

$x > 2$

$\therefore x \neq 3-\sqrt{5}$ (ext.)

$x = 3+\sqrt{5}$

2. Solve for "x": $\log_a b+\log_{a^2} b=\log_{a^3} b^x$

$\log_a b + \frac{1}{2}\log_a b = \frac{x}{3}\log_a b$

$\frac{x}{3} = 1+\frac{1}{2}$

$\frac{x}{3} = \frac{3}{2}$

$x = \frac{9}{2}$

3. Solve for "x": $\log_4 x-\log_x 16=\frac{7}{6}-\log_x 8$ [Euclid]

$\frac{\log_2 x}{\log_2 4} - \frac{\log_2 16}{\log_2 x} = \frac{7}{6} - \frac{\log_2 8}{\log_2 x}$

$\frac{\log_2 x}{2} - \frac{4}{\log_2 x} = \frac{7}{6} - \frac{3}{\log_2 x}$

$\frac{\log_2 x}{2} = \frac{7}{6} + \frac{1}{\log_2 x}$

$3\log_2 x = 7 + \frac{6}{\log_2 x}$

$3(\log_2 x)^2 = 7\log_2 x + 6$

$3(\log_2 x)^2 - 7\log_2 x - 6 = 0$

$3A^2-7A-6=0$

$(3A+2)(A-3)=0$

$A=3$ or $A=-\frac{2}{3}$

$\log_2 x = 3$

$x = 8$

or $\log_2 x = -\frac{2}{3}$

$x = 2^{-\frac{2}{3}}$

4. Determine all value(s) of "x" that satisfy the equation (find all the extraneous roots) [Euclid]

$\log_{5x+9}(x^2+6x+9)+\log_{x+3}(5x^2+24x+27)=4$

$\log_{5x+9}(x+3)^2 + \log_{x+3}(5x^2+24x+27)=4$

$2\log_{5x+9}x+3 + \log_{x+3}(5x+9)(x+3)=4$

$\frac{\log x+3}{\log 5x+9} = A$

$\frac{\log 5x+9}{\log x+3} = B$

$2\frac{\log x+3}{\log 5x+9} + 1 + \frac{\log 5x+9}{\log x+3} = 4$

$2A+B=3$ $\quad AB=1$

$\frac{2}{B}+B=3$

$2+B^2=3B$

$B^2-3B+2=0$

$(B-2)(B-1)=0$

① $\frac{\log 5x+9}{\log x+3} = 2$

$5x+9 = x^2+6x+9$

$x^2+x=0$

$x=0$ or $x=-1$

② $\frac{\log 5x+9}{\log x+3} = 1$

$5x+9 = x+3$

$4x = -6$

$x = -\frac{3}{2}$

5. If $\log_2 x$, $(1+\log_4 x)$, $\log_8 4x$ are consecutive terms of a geometric sequence, determine the possible value(s) of "x". Euclid 2009

$\log_2 x \cdot \log_8 4x = (1+\log_4 x)^2$

$A = \log_2 x$

$A \cdot \frac{1}{3}(\log_2 4 + \log_2 x) = (1+\frac{1}{2}A)^2$

$A\,\frac{1}{3}(2+A) = 1+\frac{1}{4}A^2+A$

$\frac{A^2}{3}+\frac{2}{3}A = 1+\frac{1}{4}A^2+A$

$\frac{1}{12}A^2-\frac{1}{3}A-1=0$

$A^2-4A-12=0$

$(A-6)(A+2)=0$

$\log_2 x = 6$ or $\log_2 x = -2$

$x = 64$ or $x = \frac{1}{4}$

6. Find the product of the value(s) of "m" that satisfy this equation: $(\log_2 m)^2 + \log_2 m^3 = 10$ Brilliant

$A = \log_2 m$

$A^2+3A=10$

$A^2+3A-10=0$

$(A+5)(A-2)=0$

$A=-5$ or $A=2$

$\log_2 m = -5$, $m = 2^{-5} = \frac{1}{32}$

$\log_2 m = 2$, $m = 4$

7. Determine all real numbers "x" for which $(\sqrt{x})^{\log_{10} x} = 100$ [Euclid]

$x^{\frac{1}{2}\log_{10} x} = 100$

$x^{\log_{10} x} = 10^4$

$\log_a b = \log_a c$

$\underline{b = c}$

$\log_{10} x \log_{10} x = 4$

$\log_{10} x = \pm 2$

$x = 100$ or $x = \frac{1}{100}$

8. Determine all real numbers "x" for which $(\log_{10} x)^{\log_{10}(\log_{10} x)} = 10{,}000$ [Euclid]

$\log_{10} x = A$

$A^{\log_{10}(A)} = 10^4$

$\log_{10}(A)\log_{10}(A) = 4$

$\log_{10}(A) = \pm 2$

① $\log_{10}(\log_{10} x) = 2$

$\log_{10} x = 10^2$

$x = 10^{100}$

② $\log_{10}(\log_{10} x) = -2$

$\log_{10} x = 10^{-2}$

$x = 10^{\frac{1}{100}} = \sqrt[100]{10}$

9. Challenge: Given the two equations, find the value of "n" 2003 AIME

$$\log\sin x + \log\cos x = -1 \qquad \text{and} \qquad \log(\sin x + \cos x) = 0.5(\log n - 1)$$

$\log \sin x\cos x = -1 = \log 10^{-1}$

$\log(\sin x + \cos x) = \log n^{\frac{1}{2}} - 0.5$

$\log(\sin x + \cos x) = \log\left(\frac{n^{\frac{1}{2}}}{10^{0.5}}\right)$

$\sin x\cos x = 0.1$

$\sin x + \cos x = \left(\frac{n}{10}\right)^{\frac{1}{2}}$

$\sin^2 x + \cos^2 x = 1$

$(\sin x + \cos x)^2 - 2\sin x\cos x = 1$

$\left(\left(\frac{n}{10}\right)^{\frac{1}{2}}\right)^2 - 2\times 0.1 = 1$

$\frac{n}{10} = 1.2$

$n = 12$

10. Determine the number of ordered pairs (a,b) of integers such that $\log_a b + 6\log_b a = 5$, with $2 \le a \le 2005$, and $2 \le b \le 2005$. 2005 AIME

$$\begin{cases} \log_a b + 6\log_b a = 5 \\ \log_a b \cdot \log_b a = 1 \end{cases}$$

$A = \log_a b$
$B = \log_b a$

$$\hookrightarrow \begin{cases} A + 6B = 5 \\ AB = 1 \end{cases}$$

$$A + \frac{6}{A} = 5$$

$$A^2 + 6 = 5A$$

$$A^2 - 5A + 6 = 0$$

$$(A-2)(A-3) = 0$$

$$A = 2 \text{ or } A = 3$$

① $\log_a b = 2$

$a^2 = b$

$\underbrace{a = 2 \cdots 44}_{43}$ $\quad 44^2 = 1936$
$45^2 = 2025$

② $\log_a b = 3$

$a^3 = b$

$\underbrace{a = 2 \cdot 12}_{11}$ $\quad 12^3 = 1728$
$13^3 = 2197$

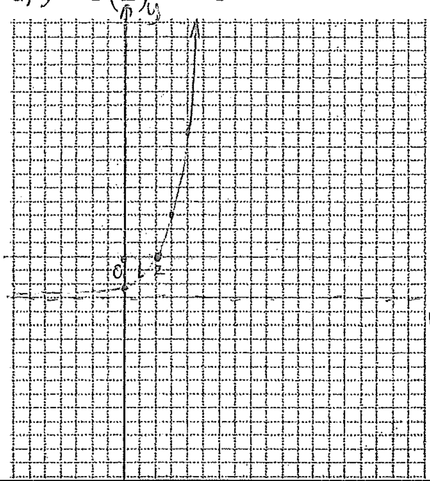
$43 + 11 = 54$

Name: Siyue Zhong Date: December 8.

Math 12 Honours: Section 5.4 Graphing Exponential & Logarithmic Equations with Transformations

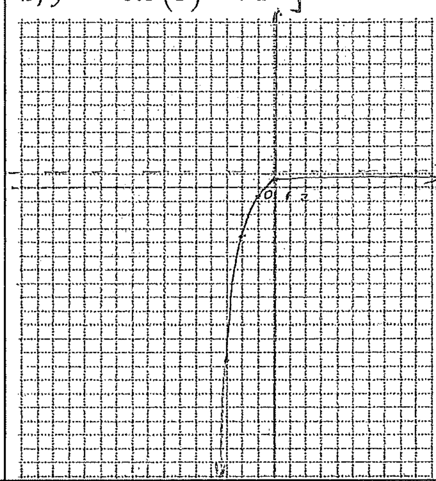
1. For each graph below, find the Y-intercept, X-intercept (if any), Domain and Range, and Asymptotes. Then graph the function with the grid provided. Be sure to label the axis on your grid.

a) $y = 3(2)^{x-2} - 3$



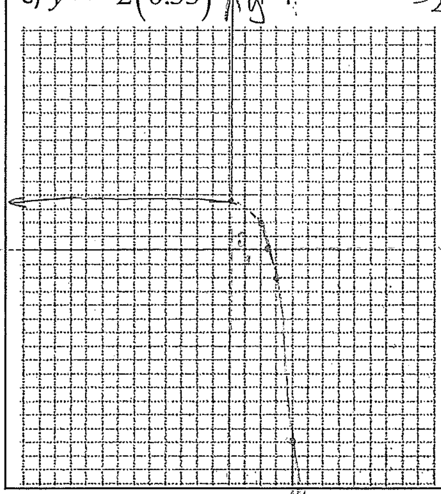
x-intercept $(2, 0)$
y-intercept $(0, -\frac{9}{4})$
asymptote: $y \neq -3$
D: $x \in \mathbb{R}$
R: $y > -3$

b) $y = -0.5(3)^{-x} + 1$



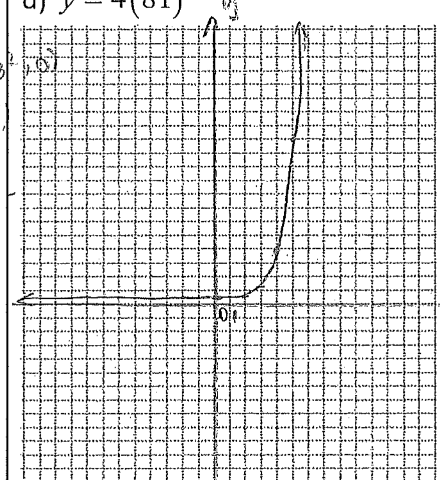
x-intercept $(-\log_3 2, 0)$
y-intercept $(0, \frac{1}{2})$
D: $x \in \mathbb{R}$
R: $y < 1$
Asymptote $y \neq 1$

c) $y = -2(0.\overline{33})^{2-x} + 4$ $-2(\frac{1}{3})^{2-x} + 4$



x intercept: $(2+\log_3 2, 0)$
y intercept: $(0, \frac{34}{9})$
D: $x \in \mathbb{R}$
R: $y < 4$
Asymptote: $y \neq 4$

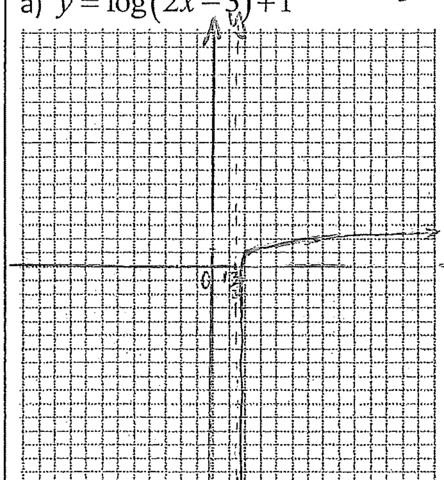
d) $y = 4(81)^{0.25x-1}$



x-intercept: none
y-intercept: $(0, \frac{4}{81})$
asymptote $y \neq 0$
D: $x \in \mathbb{R}$
R: $y > 0$

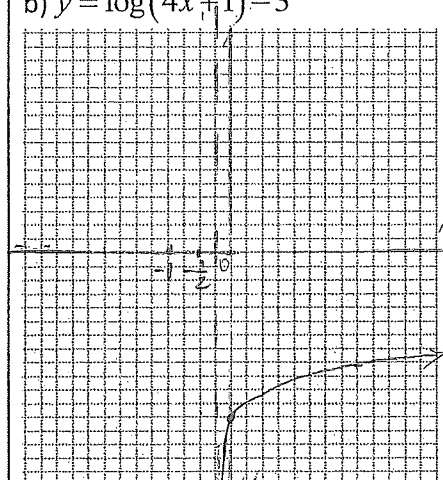
2. Graph each of the following logarithmic functions. Indicate the Domain, Range, equations of Asymptotes, and any intercepts. Be sure to label your axis on the graph:

a) $y = \log(2x-3)+1$ $2x-3 = \frac{1}{10}$ $\frac{31}{10}$



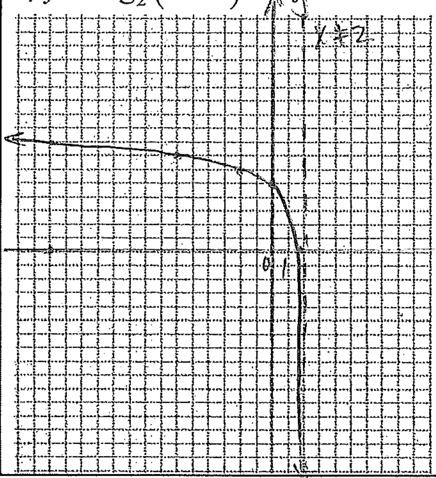
Domain: $x > \frac{3}{2}$
Range: $y \in \mathbb{R}$
y intercept: none
x intercept: $(\frac{31}{20}, 0)$
asymptote: $x \neq \frac{3}{2}$

b) $y = \log(4x+1) - 3$



Domain: $x > -\frac{1}{4}$
Range: $y \in \mathbb{R}$
y-intercept $(0, -3)$
x-intercept: $(\frac{999}{4}, 0)$
asymptote: $x \neq -\frac{1}{4}$

c) $y=\log_2(2-x)+4$

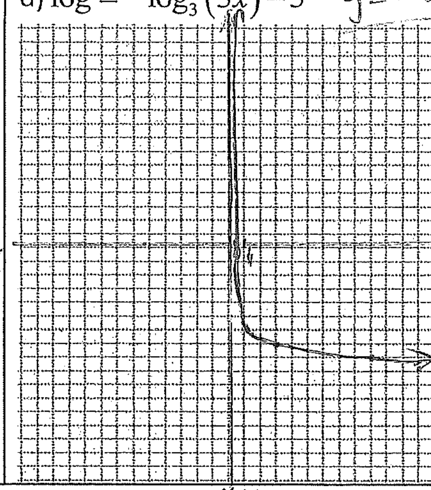


x-intercept $(\frac{31}{16}, 0)$
y-intercept $(0, 5)$
Domain $x<2$
range $y \in \mathbb{R}$
asymptote $x \neq 2$

$\log y? = -\log_3(3x)-5$ or $y=-\log_3(3x)-5$

d) $\log = -\log_3(3x)-5$

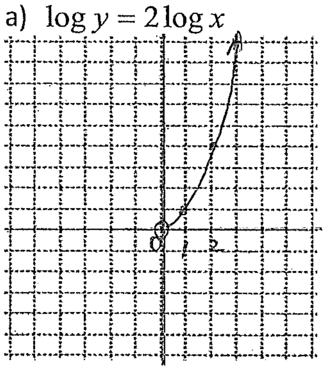
∃ graph this



x-intercept $(\frac{1}{729}, 0)$
y-intercept none
Domain $x>0$
range $y \in \mathbb{R}$
asymptote $x \neq 0$

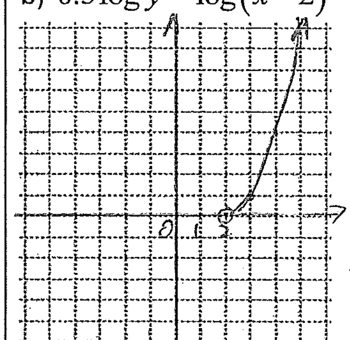
3. Graph the following functions. Indicate the domain and range:

a) $\log y = 2\log x$



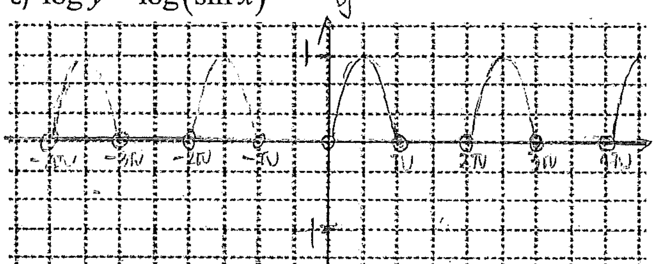
Domain : $x>0$
Range : $y>0$

b) $0.5\log y = \log(x-2)$



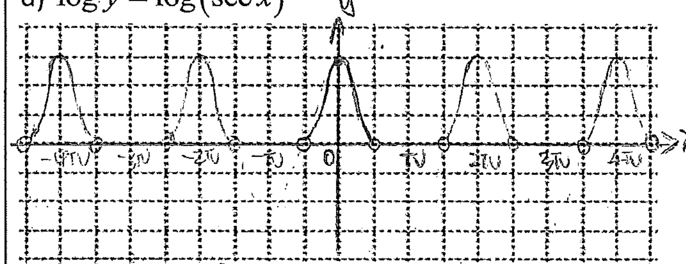
domain : $x>2$
Range : $y>0$

c) $\log y = \log(\sin x)$



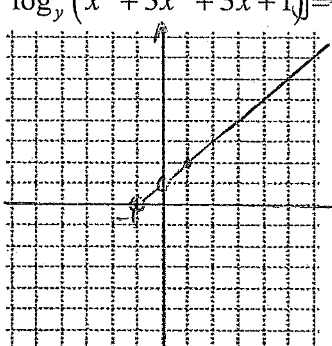
Domain $2n\pi < x < (2n+1)\pi \quad n \in \mathbb{Z}$.
Range $0 < y \leq 1$

d) $\log y = \log(\sec x)$



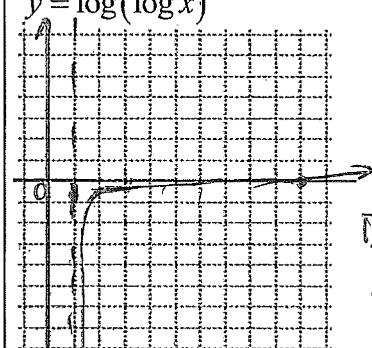
Domain $-\frac{\pi}{2}+n\pi < x < n\pi+\frac{\pi}{2}$
Range $1 < y \leq 1$

$\log_y(x^3+3x^2+3x+1) = 3$



$\log_y(x+1)^3 = 3$
$(x+1)^3 = y^3$
Similar to $y=x+1$
Range : $y>0$, $y \neq 1$
Domain $x>-1 \quad x \neq 0$

$y=\log(\log x)$



$x>0$
$\log x > 0$
$x>1$
Domain $x>1$
Range : $y \in \mathbb{R}$.

4. Given that $f(x)=\log_3(x+2)-4$, find $f^{-1}(x)$, the inverse function of $f(x)$

$\log_3(y+2)-4 = x$
$\log_3(y+2) = x+4$
$3^{x+4} = y+2$
$y = 3^{x+4}-2$.

5. What transformation is required to go from $y = \log x$ to $y = \log\left(\frac{1}{x}\right)$?

$y = -\log x$

vertical reflection

6. What transformation is required to go from $y = \log_3 x$ to $y = \log_3 \frac{4}{x}$?

$y = \log_3 \frac{4}{x} = \log_3 4 - \log_3 x$

Vertical reflection then vertical shift up $\log_3 4$ unit.

7. Are the following graphs the same? Yes or NO? Explain:

a) $y = 4(0.5)^x$ and $y = 4(2)^{-x}$

Yes

$4(0.5)^x = 4(\frac{1}{2})^x = 4 \cdot 2^{-x}$

b) $y = 24(0.5)^{2-x}$ and $y = 6(2)^x$

$24(0.5)^{2-x} = 6 \times 4 \times (\frac{1}{2})^2 (\frac{1}{2})^{-x}$

$= 6 \times 2^x$

Yes.

8. What is the inverse function of $f(x) = 3(5)^{x-2}$? What are the domain, range, x-intercepts, and Y-intercepts of both $f(x)$ and $f^{-1}(x)$? What patterns do you notice?

$f(x)$ domain $x \in \mathbb{R}$ x intercept: none

range $y > 0$ y-intercept: $(0, \frac{3}{25})$

$y = 3(5)^{x-2}$

Switch(x, y) $x = 3(5)^{y-2}$

$x = \frac{3}{25} 5^y$

$\frac{25}{3} x = 5^y$

$\log_5 \frac{25}{3} x = y$

$f^{-1}(x)$ domain $x > 0$

range

domain and range swapped.

x and y intercepts swaped.

9. Find the coordinates of the points of intersection of the graphs: $y = \log_{10}(x-2)$ and $y = 1 - \log_{10}(x+1)$

(Euclid)

$y = \log_{10}(x-2)$

$y = 1 - \log_{10}(x+1)$

$\log_{10}(x-2) = \log_{10}\left(\frac{10}{x+1}\right)$

$(x-2)(x+1) = 10$

$x^2 - x - 2 = 10$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ or $x = -3$

but $x - 2 > 0$ $x + 1 > 0$

$x \neq -3$ ext.

$x = 4$

$y = \log_{10} 2$

$(4, \log_{10} 2)$

10. Determine all the points where the two functions intersect: $y = \log_{10} x^4$ and $y = (\log_{10} x)^3$ (Euclid)

$\log_{10} x^4 = (\log_{10} x)^3$

$A = \log_{10} x$

$4A = A^3$

$A = 0$

$A = \pm 2$

1. $\log_{10} x = 1$

$x = 10$

2. $\log_{10} x = -2$

$x = \frac{1}{100}$

3. $\log_{10} x = 2$

$x = 100$

$x = 10$ or 100 or $\frac{1}{100}$

$(10, 4)$

$(100, 8)$ $(\frac{1}{100}, -8)$

11. If $\log(a+b) = x$ and $\log(a^2 - ab + b^2) = y$, then what is the value of $\sqrt[4]{a^3 + b^3}$ in terms of "x" and "y"?

$\log(a+b) + \log(a^2 - ab + b^2) = x + y$

$\log(a^3 + b^3) = x + y$

$a^3 + b^3 = 10^{x+y}$

$\sqrt[4]{a^3 + b^3} = \sqrt[4]{10^{x+y}}$

12. Solve the inequality

a) $\log_4(2x-3)>2$

$2x-3>16$

$x>\frac{19}{2}$

b) $\log_{\frac{1}{4}} x>5$

$0<x<\left(\frac{1}{4}\right)^5$

$0<x<\frac{1}{1024}$

13. What is the domain of the following functions:

i) $y=\log_{0.5}(\log_5 x)$

$x>0$

$\log_5 x>0$

$x>1$

ii) $y=\log_{0.5}\left(\log_5\left(\log_{\frac{1}{3}} x\right)\right)$

$\log_{\frac{1}{3}} x>1$

$x>0$

$0<x<1$

14. Solve for "x": $\log_4 x-\log_x 16=\frac{7}{6}-\log_x 8$ (Euclid)

$\frac{\log x}{\log 4}-\frac{\log 2}{\log x}=\frac{7}{6}$

$(\log x)^2-\log 2\log 4=\frac{7}{6}\log 4\log x$

$\log_2 x=A$

$A^2-\frac{7}{3}A-2=0$

$3A^2-7A-6=0$

$(3A+2)(A-3)=0$

$A=-\frac{2}{3}$ or $A=3$

$x=8$ or $x=\frac{1}{\sqrt[3]{4}}$

15. Find all the values of "x" such that: $\log_{2x}\left(48\sqrt[3]{3}\right)=\log_{3x}\left(162\sqrt[3]{2}\right)$ (Euclid)

$\log_{2x}48\sqrt[3]{3}=\log_{3x}162\sqrt[3]{2}=m$

$(2x)^m=48\sqrt[3]{3}=2^4 3^{\frac{4}{3}}$

$(3x)^m=162\sqrt[3]{2}=3^4 2^{\frac{4}{3}}$

$\left(\frac{2}{3}\right)^m=\frac{2^{\frac{8}{3}}}{3^{\frac{8}{3}}}$

$m=\frac{8}{3}$

$\log_{2x}2^4 3^{\frac{4}{3}}=\frac{8}{3}$

$(2x)^{\frac{8}{3}}=2^4 3^{\frac{4}{3}}$

$2x=2^{\frac{3}{2}}3^{\frac{1}{2}}$

$x=\sqrt{6}$

16. Determine all real numbers "x" for which: $2\log_2(x-1)=1-\log_2(x+2)$ (Euclid)

$\log_2(x-1)^2(x+2)=1$

$(x-1)^2(x+2)=2$

$(x^2-2x+1)(x+2)=2$

$x^3-\cancel{2x^2}+x+\cancel{2x^2}-4x=0$

$x^3-3x=0$

$x-1>0$ $x+2>0$

$x>1$ $x>-2$

$x\neq 0$

$x^2-3=0$

$x=\sqrt{3}$

17. Determine all pairs of angles (x,y) with $0° \le x < 180°$ and $0° \le y < 180°$ that satisfy the following systems of equations: (Euclid)

$$\log_2(\sin x \cos y) = -\frac{3}{2} \quad \text{and} \quad \log_2\left(\frac{\sin x}{\cos y}\right) = \frac{1}{2}$$

$\begin{cases} \sin x \cos y = 2^{-\frac{3}{2}} \\ \frac{\sin x}{\cos y} = 2^{\frac{1}{2}} \end{cases}$

$\sin x \cos y = \frac{1}{2\sqrt{2}}$

$\frac{\sin x}{\cos y} = \sqrt{2}$

$\cos y \neq 0$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

① $\begin{cases} \sin x = \frac{1}{\sqrt{2}} \\ \cos y = \frac{1}{2} \end{cases}$

$x = 45°$ or $135°$

$y = 60°$ or $120°$

(45, 60) (45, 120) (135, 60) (135, 120)

② $\sin x = -\frac{1}{\sqrt{2}}$

$\cos y = -\frac{1}{2}$

x, y are not between 0 and 180°.

18. Determine all pairs (a,b) of real numbers that satisfy the following systems of equations: Give your answer in simplified exactd form: (Euclid)

$$\sqrt{a} + \sqrt{b} = 8 \quad \text{and} \quad \log_{10} a + \log_{10} b = 2$$

$A = \sqrt{a}$

$B = \sqrt{b}$

$A + B = 8$

$\log A^2 + \log B^2 = 2$

$\log A^2B^2 = \log 100$

$A^2B^2 = 100$

$\begin{cases} A+B = 8 \\ AB = \pm 10 \end{cases}$

$A \ge 0 \quad B \ge 0$

$\begin{cases} A+B = 8 \\ AB = 10 \end{cases}$

$A(8-A) = 10$

$A^2 - 8A + 10 = 0$

$A = \frac{8 \pm \sqrt{64-40}}{2}$

$= \frac{8 \pm 2\sqrt{6}}{2}$

$= 4 \pm \sqrt{6}$

$\begin{cases} A = 4+\sqrt{6} \\ B = 4-\sqrt{6} \end{cases}$ or $\begin{cases} A = 4-\sqrt{6} \\ B = 4+\sqrt{6} \end{cases}$

$\begin{cases} \sqrt{a} = 4+\sqrt{6} \\ \sqrt{b} = 4-\sqrt{6} \end{cases}$

$\begin{cases} a = 22+8\sqrt{6} \\ b = 22-8\sqrt{6} \end{cases}$ or $\begin{cases} a = 22-8\sqrt{6} \\ b = 22+8\sqrt{6} \end{cases}$

19. Solve for "x" $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020}$. (AIME)

$\frac{20}{x}\frac{\log 3}{\log 2} = \frac{2020}{x+3}\frac{\log 3}{\log 2}$

$\frac{20}{x} = \frac{2020}{x+3}$

$20x + 60 = 2020x$

$2000x = 60$

$x = \frac{3}{100}$

Problem

The value of x that satisfies $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Solution

Let $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020} = n$. Based on the equation, we get $(2^x)^n = 3^{20}$ and $(2^{x+3})^n = 3^{2020}$. Expanding the second equation, we get $8^n \cdot 2^{xn} = 3^{2020}$. Substituting the first equation in, we get $8^n \cdot 3^{20} = 3^{2020}$, so $8^n = 3^{2000}$. Taking the 100th root, we get $8^{\frac{n}{100}} = 3^{20}$. Therefore, $(2^{\frac{3}{100}})^n = 3^{20}$, and using the our first equation($2^{xn} = 3^{20}$), we get $x = \frac{3}{100}$ and the answer is $\boxed{103}$. ~rayfish

Name: Siyue Zhang. Date:

Math 12 Honours: section 5.5 Applications of Logarithms and Exponential Functions:

1. Suppose you invested \$1000 at 5% compounded annually, how many years will it take your investment to grow to i) \$5000? Ii) \$10,000 iii) \$100,000

i)
$$1000\times\left(1+\frac{5\%}{1}\right)^x = 5000$$
$$(1+5\%)^x = 5$$
$$1.05^x = 5$$
$$x = \log_{1.05}5$$
$$x = 32.9869$$

ii)
$$1000(1+5\%)^x = 10000$$
$$1.05^x = 10$$
$$x = \log_{1.05}10$$
$$x = 47.19$$

iii)
$$1000(1+5\%)^x = 100000$$
$$(1.05)^x = 100$$
$$x = \log_{1.05}100$$
$$x = 94.39$$

2. Calculate the number of years it would take an investment to double for each of the following:

i) 10% compounded semi-annually
$$P\left(1+\frac{10\%}{2}\right)^{2x} = 2P$$
$$(1.05)^{2x} = 2$$
$$2x = \log_{1.05}2$$
$$x = \frac{1}{2}\log_{1.05}2$$
$$x = 7.1$$

ii) 10% compounded daily
$$\left(1+\frac{10\%}{365}\right)^{365x} = 2$$
$$365x = \log_{1+\frac{10\%}{365}}2$$
$$x = 6.93$$

iii) 8% compounded weekly
$$\left(1+\frac{8\%}{52}\right)^{52x} = 2$$
$$52x = \log_{\left(1+\frac{8\%}{52}\right)}2 \qquad x = 8.67$$

iv) 8% compounded monthly
$$\left(1+\frac{8\%}{12}\right)^{12x} = 2$$
$$12x = \log_{\left(1+\frac{8\%}{12}\right)}2$$
$$x = 8.69$$

3. In general, for "k%" interest, how many years does an investment take to double? Triple?

1) double
$$(1+k\%)^x = 2$$
$$x = \log_{1+k\%}2$$
$\log_{1+k\%}2$ years to double

2) triple
$$(1+k\%)^x = 3$$
$$x = \log_{1+k\%}3$$
$\log_{1+k\%}3$ years to triple.

4. A 5% investment compounded daily is equivalent to what interest rate compounded annually?
$$\left(1+\frac{5\%}{365}\right)^{365} = (1+k\%)^1$$
$$k\% = 5.13\%$$
5.13% interest rate compounded annually

5. Suppose you invest \$5000 each year into an investment that gives a 10% return, how much will your portfolio be after 25 years?
$$5000\times(1+10\%)^{25} + 5000(1+10\%)^{24} + \cdots + 5000(1+10\%)^1$$
$$= 5000(1.1^{25} + 1.1^{24} + 1.1^{23} + \cdots + 1.1)$$
$$= 5000\left(\frac{1.1^{26} - 1.1}{1.1 - 1}\right)$$
$$= 540908.8269.$$

6. The half life of sodium-24 is 15.5 hours. Suppose a hospital buys a 100mg sample of sodium-24, how much of the sample will be left after 72 hours?

$$100 \times \left(\frac{1}{2}\right)^{\frac{72}{15.5}} = 3.996 \text{ mg.}$$

7. How long will it take a 100mg sample of sodium-24 to decompose to 1mg?

$$100 \times \left(\frac{1}{2}\right)^{\frac{x}{15.5}} = 1$$

$$\left(\frac{1}{2}\right)^{\frac{x}{15.5}} = \frac{1}{100}$$

$$\log_{\frac{1}{2}} \frac{1}{100} = \frac{x}{15.5}$$

$$x = 15.5 \log_{\frac{1}{2}} \frac{1}{100}$$

$$x = 102.9798$$

It will take around 102.9798 hours.

8. Iodine-129 is a dangerous substance in radioactive waste with a half life of 16million years. How long will it take 1,000g of Iondine-129 to decompose to 1g?

$$1000 \cdot \left(\frac{1}{2}\right)^{\frac{x}{16}} = 1$$

$$\left(\frac{1}{2}\right)^{\frac{x}{16}} = \frac{1}{1000}$$

$$\log_{\frac{1}{2}} \frac{1}{1000} = \frac{x}{16}$$

$$\frac{x}{16} = \log_{\frac{1}{2}} \frac{1}{1000}$$

$$x = 16 \log_{\frac{1}{2}} \frac{1}{1000}$$

$$x = 159.45$$

It will take around 159.45 million years.

9. A 500mg radioactive substance decomposed to 20mg in 25 years. What is the half life of the radioactive substance?

$$500 \times \left(\frac{1}{2}\right)^{\frac{25}{x}} = 20$$

$$\left(\frac{1}{2}\right)^{\frac{25}{x}} = \frac{1}{25}$$

$$\log_{\frac{1}{2}} \frac{1}{25} = \frac{25}{x}$$

$$x = \frac{25}{\log_{\frac{1}{2}} \frac{1}{25}}$$

$$x = 5.38$$

The half-life is about 5.38 years.

10. A large fossil with 25mg of carbon 14 was found that originally contained 3000mg of carbon 14. If the half life of carbon 14 is 5700 years, then how old is the fossil?

$$3000 \times \left(\frac{1}{2}\right)^{\frac{x}{5700}} = 25$$

$$\left(\frac{1}{2}\right)^{\frac{x}{5700}} = \frac{1}{120}$$

$$\frac{x}{5700} = \log_{0.5} \frac{1}{120}$$

$$x = 5700 \log_{0.5} \frac{1}{120}$$

$$x = 39369.28$$

The fossil is around 39369.28 years old.

11. 5% of a substance decays in 100 days. What is the half life of the substance?

$$100\% \times \left(\frac{1}{2}\right)^{\frac{100}{x}} = 95\%$$

$$\left(\frac{1}{2}\right)^{\frac{100}{x}} = \frac{19}{20}$$

$$\frac{100}{x} = \log_{\frac{1}{2}} \frac{19}{20}$$

$$x = \frac{100}{\log_{\frac{1}{2}} \frac{19}{20}}$$

$$x = 1351.34$$

The half life is around 1351.34 days.

12. A swarm of locusts can multiply in population by 5 times every 6 weeks. How long will it take a cluster of 5000 locusts to grow to 1 million?

$$5000 \times 5^{\frac{x}{6}} = 10^6$$

$$5^{\frac{x}{6}} = 200$$

$$\frac{x}{6} = \log_5 200$$

$$x = 6\log_5 200$$

$$x = 19.75$$

It will take around 19.75 week

13. The amount of intensity of an earthquake is measured by the amount of ground motion recorded on a seismograph. An increase in 1unit magnitude on the Ritcher scale represents a 10 fold in intensity. How many times more 'intense' is an earthquake of magnitude 9 than an earthquake of 5.5?

$$10^{9-5.5} = 3162.27766$$

3162.27766 times more intense.

14. For each increase of 1 unit in magnitude, earthquakes are 32 times more powerful in the amount of energy. An earthquake of magnitude of 1 has about $794{,}328J$ of energy. An earthquake of magnitude 2 has about $25{,}118{,}864J$ of energy. An earthquake of 2.5 magnitude has 141,253,754J of energy. How many much energy is in an earthquake with an magnitude of 6.5?

$$141253754 \times 32^{6.5-2.5} = 1.48 \times 10^{14} J$$

around 1.48×10^{14} Joules.

15. In 1976, an earthquake in Italy released approximately 10^{14} joules of energy. What is the magnitude of that earthquake?

$$794328 \times 32^{x-1} = 10^{14}$$

$$32^{x-1} = 125892578.4$$

$$x-1 = \log_{32} 125892578.4$$

$$x-1 = 5.38$$

$$x = 6.38$$

The magnitude is around 6.38.

16. The loudness of sound is measured in decibals (db). Every increase in 10db represents a 10 fold increase in loudness. Going from 10db to 30db, an increase in 20db, is 10^2 times more loud. A regular conversation is about 55db. Listening to a Rock concert is about 110db. How many times louder is the Rock concert than a normal conversation?

$$10^{\frac{110-55}{10}} = 316227.766$$

it's around 316227.766 times louder.

17. A soft whisperer speaks at about 30db. Getting in a yelling match with Cheong is about a thousand times louder. How many db would Cheong be shouting at?

$$10^{\frac{x-30}{10}} = 1000$$

$$\frac{x-30}{10} = 3$$

$$x = 60 \text{ db}$$

Mr. Cheong would be shouting at 60 db.

18. In chemistry, the pH scale measures acidity or alkalinity of a solution. pH ranges from 0 (acidic) to 14 (alkalinity), with 7 being neutral. For each increase in pH, there is a 10 fold increase in alkalinity. For each decrease in pH, there is a 10 fold increase in acidity. Tomato juice has a pH of 4.2 and lemon juice is about 1.8 in pH. How much more acidic is lemon juice than tomato juice?

$$10^{4.2-1.8} = 251.1886$$

It's around 251.1886 times more acidic.

19. A dishwasher soap has a pH of 12.8. Baking soda is about 9.3 in pH. How much more alkaline is the dishwater soap than baking soda?

$$10^{12.8-9.3} = 3162.27766$$

It's around 3162.27766 times more alkaline.

20. The pH values of milk ranges from 6.4 to 7.6. Pure water has a pH value of 7.0. How much more acidic is milk with a pH of 6.4 than pure water?

$$10^{7-6.4} = 10^{0.6} = 3.98$$

It's around 3.98 times more acidic.

21. How much more alkaline is milk with a pH of 7.6 than pure water?

$$10^{7.6-7} = 10^{0.6} = 3.98$$

It's around 3.98 times more alkaline.

22. The total amount of arable land in th world is about $3.2\times10^{9}\,ha$. An hectare of land is about 100m by 100m. 0.4 ha of land is enough to grow food for one person. World population in 2021 is 7.874billion, growing at 1.1% annually. When will the demand for arable land exceed the supply available?

$$7.874\times\cancel{10^9}\times(1+1.1\%)^x\times0.4 = 3.2\times\cancel{10^9}$$

$$(1+1.1\%)^x = 1.016002032\ldots$$

$$x = \log_{1.011}1.016\ldots$$

$x = 1.45$

around 1.45 years later from 2021 which is around 2022 year May.

23. If we are able to reduce the growth rate of the world population by 0.5 and increase the productivity of food production by 200%, then how long will we have until the demand for arable land exceed supply?

$$7.874\times\cancel{10^9}\times\left(1+\frac{1.1\%}{2}\right)^x\times\frac{0.4}{3} = 3.2\times\cancel{10^9}$$

$$(1+0.55\%)^x = \frac{3.2\times3}{0.4\times7.874}$$

$$1.0055^x = 3.048\ldots$$

$$x = 203.19$$

We have around 203.19 years.

14. When $p=\sum_{k=1}^{6} k\ln k$, the number e^p is an integer. What is the largest power of 2 that is a factor of e^p ? AMC 12B

$$P = \ln 1 + 2\ln 2 + 3\ln 3 + 4\ln 4 + 5\ln 5 + 6\ln 6$$
$$= \ln 1 \cdot 2^2 \cdot 3^3 \cdot 4^4 \cdot 5^5 \cdot 6^6$$

$$e^P = 1 \cdot 2^2 \cdot 3^3 \cdot 4^4 \cdot 5^5 \cdot 6^6$$
$$= 2^2 \cdot 2^8 \cdot 2^6 \cdot 3^6 \cdot 3^3 \cdot 5^5$$
$$= 2^{16} \cdot 3^9 \cdot 5^5$$

2^{16}

15. Challenge: What is the value of $\lim_{xy\to 1}\left(\frac{\ln x}{\ln y}+\frac{\ln y}{\ln x}\right)=?$

$$= \lim_{xy\to 1}\left(\frac{\ln xy}{\ln y} + \frac{\ln xy}{\ln x} - 2\right) = -2$$

16. Evaluate: $\sum_{n=2}^{\infty}\frac{1}{n(n-1)^3}$, given Euler's beautiful result that: $1+\frac{1}{2^2}+\frac{1}{3^2}+....+\frac{1}{n^2}+....=\frac{\pi^2}{6}$

[Math Circles]

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$$

$$\sum_{(n-1)=1}^{\infty}\frac{1}{(n-1)^2} = \frac{\pi^2}{6}$$

$$\sum_{n=2}^{\infty}\frac{1}{(n-1)^2} = \frac{\pi^2}{6}$$

$$\sum_{n=2}^{\infty}\frac{1}{n(n-1)^3} = \sum_{n=2}^{\infty}\frac{A}{n} + \frac{B}{(n-1)} + \frac{C}{(n-1)^2} + \frac{D}{(n-1)^3}$$

$$(n-1)^3A + n(n-1)^2B + n(n-1)C + nD = 1$$

$$A(n^3 - 3n^2 + 3n - 1) + (n^3 - 2n^2 + n)B + (n^2 - n)C + nD =$$

$A = -B$

$+3A - 2B + C = 0$

$3A + B - C + D = 0$

$-A = 1$

$A = -1$

$B = 1$

$C = -1$

$D = 1$

$$\sum_{n=2}^{\infty}\frac{1}{n(n-1)^3} = \sum_{n=2}^{\infty}\underbrace{\boxed{\frac{-1}{n}+\frac{1}{n-1}}}_{\text{telescoping}} - \frac{1}{(n-1)^2} + \frac{1}{(n-1)^3}$$

$$= 1 - \frac{\pi^2}{6} + \zeta(3)$$

↳ Apéry constant

$$= \sum_{n=1}^{\infty}\frac{1}{n^3}$$

Name:______________________________ Date:________________________________

Math 12 Honours: section 5.6 Natural Logarithms and e

1. When should you use the formula $A = P\times e^{rxt}$ versus the final equation: $F = I\times N^{\frac{A}{L}}$

when continous growing or decaying, I should use formula $A=Pe^{rt}$; when the times of growth is limited, I should use $F=I\times N^{\frac{A}{L}}$.

2. When a population is $\boxed{\text{decreasing}}$ continuously at a rate of 3.5%, then what should the value of "r" be in the equation $A = P\times e^{rt}$?

r should be negative $r = -3.5\%$

$\lim_{n\to\infty} P(1-\frac{3.5\%}{n})^{nt}$

3. What is the value of the irrational number "e"? How can it be derived?

$e = \lim_{n\to\infty}(1+\frac{1}{n})^n = 2.71818\cdots$

4. Simplify each of the following into a single exponent:

a) $\frac{2}{e^{-x}} = 2e^{x}$	b) $(e^x)^4 = e^{4x}$	c) $e^{1-x}e^{3x} = e^{2x+1}$
d) $e^x e^{-2x} = e^{-x}$	e) $3e^{2x}(1-6e^{-3x}) = 3e^{2x} - 18e^{-x}$ $= 3e^{2x} - \frac{18}{e^x}$	f) $4e^{3x-2}(2-3e^{2x})$ $= 8e^{3x-2} - 12e^{5x-2}$

5. Evaluate each of the following without using a calculator

a) $e^{2\ln 4} = 4^2 = 16$	b) $\ln e^3 = 3$	c) $-3\ln e = -3$
d) $e^{-4\ln 4} = 4^{-4} = \frac{1}{64}$ need a calculator	e) $\ln\sqrt[4]{e^5} = \frac{5}{4}$	f) $\ln\left(\frac{1}{e}\right) = -1$
g) $\ln(1+e) = 1.3132616\cdots$	h) $e^{\ln 0} = 0$ but is ln 0 an undefined value? Correction: $e^{\ln 0} =$ undefined	i) $\ln 3 + 2\ln 4 - \ln 48$ $= \ln\frac{3\times 4^2}{48}$ $= 0$

6. Reduct the following to lowest term

a) $e^{-2\ln 3+3\ln 2}$ $= e^{-2\ln 3}\cdot e^{3\ln 2}$ $= (3)^{-2}\cdot 2^3 = \frac{8}{9}$	b) $e^{-\ln\left(\frac{1}{e}\right)}$ $= e$	c) $\ln(3^{-e}e^3)$ $= \ln 3^{-e} + \ln e^3$ $= -e\ln 3 + 3$
d) $\ln\left(\frac{\sqrt{\pi}}{e}\right) = \ln\sqrt{\pi} - 1$ $= \frac{1}{2}\ln\pi - 1$	e) $\ln e^{2\ln e}$ $= \ln e^2$ $= 2$	f) $e^{\ln 3^{(\ln 4)}}$

7. Solve for "x"

a) $e^{3x}=4$ $3x = \ln 4$ $x = \frac{\ln 4}{3}$	b) $\ln x = 11$ $e^{11} = x$ $x = e^{11}$	c) $\ln(3x-2)=4$ $3x-2 = e^4$ $x = \frac{e^4+2}{3}$
d) $\ln\left(e^{4-x}\right)=6$ $4-x=6$ $x=-2$	e) $e^{5-3x}=4$ $5-3x=\ln 4$ $x=\frac{5-\ln 4}{3}$	f) $\ln x = \ln 11+\ln 6-\ln 3$ $\ln x = \ln 22$ $x = 22$
g) $\ln(\ln x)=4$ $\ln x = e^4$ $x = e^{(e^4)}$	h) $e^{e^{2x}}=4$ $x = \frac{\ln(\ln 4))}{2}$ $e^{2x} = \ln 4$ $2x = \ln(\ln 4))$	i) $\frac{\ln\sqrt{x}}{2}=3$ $\ln\sqrt{x}=6$ $\ln x = 12$ $x = e^{12}$
j) $\ln(4x-1)=\frac{\log 10}{\log e}$ $\ln(4x-1)=\ln 10$ $4x-1=10$ $x = \frac{11}{4}$	k) $\left(e^{\ln 4x}\right)^2=4$ $(4x)^2=4$ $4x = \pm 2$ $x = \pm\frac{1}{2}$ but $x>0$ $x = \frac{1}{2}$	l) $\ln x^e = 1$ $e \ln x = 1$ $\ln x = \frac{1}{e}$ $x = e^{\frac{1}{e}}$
m) $(\ln x)^2-\frac{\log x}{\log e}+6=0$ $(\ln x)^2-\ln x+6=0$ $\ln x =$ no solution $\Delta = (-1)^2-4\times 6<0$	n) $e^x-6e^{-x}=1$ $e^x = A$ $A-\frac{6}{A}=1$ $A^2-6=A$ $A^2-A-6=0$ $(A+2)(A-3)=0$ $A=-2$ or $A=3$ $e^x>0$ $x=\ln 3$ $e^x=3$	o) $(\ln x)^2+(\ln x)=2$ $\ln x>0$ $\ln x = 1$ $x = e$
p) $2\ln x = \ln(4x+5)$ $\ln x^2 = \ln(4x+5)$ $x^2=4x+5$ $x^2-4x-5=0$ $(x+1)(x-5)=0$ $x>0$ $x=5$	q) $e^x+4e^{-x}=5$ $e^x=A$ $A+\frac{4}{A}=5$ $A^2+4-5A=0$ $(A-1)(A-4)=0$ $e^x=1$ $x=0$ or $e^x=4$ $x=\ln 4$	r) $e^{\ln x^3-1}=12$ $e^{\ln x^3}=12e$ $x^3=12e$ $x=\sqrt[3]{12e}$

8. Express the following as a single logarithm:

a) $\frac{1}{3}\ln x+2\ln(6x+5)$ $=\ln\sqrt[3]{x}\cdot(6x+5)^2$	b) $3\ln x-\ln\left(x^2-1\right)+\ln\left(x^2-1\right)$ $=\ln x^3$ $=3\ln x$
c) $4\ln x+5\ln(2-x)-2\ln(1+x)$ $=\ln\frac{x^4(2-x)^5}{(1+x)^2}$	d) $\frac{2}{3}\ln x-3\ln\left(x^2-2x-8\right)$ $=\ln\left(\frac{x^{\frac{2}{3}}}{(x^2-2x-8)^3}\right)$

9. The relationsip between the elasped time "t", in hours, since Jack took his first does of medication, and the amount of medication M(t), in mg, in his bloodstream is modelled by the following function below.

$M(t) = 30 \times e^{-0.8t}$

I) How much medication will Jack have in his bloodstream after 3 hours?

$M(3) = 30 \times e^{-0.8 \times 3} = 30 \times e^{-2.4} \approx 2.7215 \ldots$

II) How many hours will it take for Jack to have 1mg left in his bloodstream?

$30 \times e^{-0.8t} = 1$

$e^{-0.8t} = \frac{1}{30}$

$-0.8t = \ln \frac{1}{30}$

$t = -\frac{\ln \frac{1}{30}}{0.8}$

$\approx 4.251497 h$

10. The amount of money Dave has in his investment is given by the formula: $A = Pe^{rt}$. If He invests \$5000 at 2.5% interest, compounded continueously, how long will it take to double his investment?

$2\cancel{P} = \cancel{P}e^{2.5\% t}$

$e^{2.5\% t} = 2$

$2.5\% t = \ln 2$

$t = \frac{\ln 2}{0.025}$

$= 27.726$ years.

11. A radio-active substance has a half life of 2500 years. What is the equation for the amount of this substance after "t" years in the form of $A = Pe^{rt}$?

$A = Pe^{rt}$

$Pe^{r \cdot 2500} = \frac{1}{2}P$

$e^{r \cdot 2500} = \frac{1}{2}$

$2500r = \ln \frac{1}{2}$

$r = -\frac{\ln 2}{2500}$

$A = Pe^{-\frac{\ln 2}{2500} \cdot t}$

12. TD bank offers an GIC that gives annual interest of 1.5% compounded monthly. What is the equivalent interest rate if the interest is compounded continuously?

$A = Pe^{rt}$

$A = P(1 + \frac{1.5\%}{12})^{12t}$

$e^{r} = (1 + \frac{1.5\%}{12})^{12}$

$r = 12 \ln (1 + \frac{1.5\%}{12})$

$\approx 1.249 \times 10^{-3}$

$A = Pe^{1.249 \times 10^{-3} \cdot t}$

13. Each year, Jason's parents contributes \$2500 into his RESP account, then govt will match it with \$500. Suppose the RESP is invested in a fund that gives 8% return annually, compounded continuously, starting when Jason was born, how much will he have in the account when he turns 18?

$A = 3000 \times (1.08)^{18} + 3000 \times (1.08)^{17} + 3000 (1.08)^{16} + \cdots$

$+ 3000 \times (1.08)^{1}$

$= \frac{3000 \times (1.08)^{19} - 3000 (1.08)}{1.08 - 1} \approx 121338.7897$